FINAL REPORT(*)

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OPTIMUM LIFTING BODIES IN NEWTONIAN FLOW

by

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1. INTRODUCTION

The object of this investigation is a general analysis of optimum lifting bodies in hypersonic flow. Physically, Newtonian theory is employed. Mathematically, the methods of the calculus of variations are used in order to optimize the lift-to-drag ratio of a lifting body for given constraints imposed on the length, the thickness, and the volume. This quantity is important in that the range and the maneuverability of a hypersonic cruise vehicle, a hypersonic glide vehicle, and a reentry vehicle increase linearly with it. Since flat-top bodies are naturally suited to produce high lift-to-drag ratios at hypersonic speeds, particular attention is devoted to these bodies under the assumption that the free-stream velocity is parallel to the flat top.

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2. DIRECT METHODS (Refs. 1 and 7)

An investigation of the lift-to-drag ratio attainable by a slender, flat-top, homothetic body is presented under the assumptions that the longitudinal contour is a power law and the transversal contour is semielliptical or triangular. Analytical expressions are derived relating the drag, the lift, and the lift-to-drag ratio to the geometry of the configuration.

The effect of the thickness ratio, the power law exponent, and the elongation ratio of the cross section on the lift-to-drag ratio is investigated. It is shown that the optimum thickness ratio is such that the friction drag is one-third of the total drag; the optimum power law exponent is one; and the optimum elongation ratio is such that the configuration is body-like if the cross section is semielliptical and wing-like if the cross section is triangular.

For a friction coefficient $C_f = 10^{-3}$, the maximum attainable lift-to-drag ratio is $E = 3.67$ with a semielliptical cross section and $E = 5.29$ with a triangular cross section. Thus, bodies of triangular cross section are considerably more efficient than bodies of semielliptical cross section.

3. DIRECT METHODS (Ref. 4)

The problem considered in Ref. 1 is investigated once more with reference to the class of slender, flat-top bodies whose longitudinal contour is a power law and whose transversal contour is semicircular.

Constrained configurations are considered, that is, conditions are imposed on the length, the thickness, the volume, and the position of the center of pressure. For each combination of constraints, an appropriate similarity parameter is introduced, and the optimum power law exponent, thickness ratio, and lift-to-drag ratio are determined as functions of the similarity parameter.

4. INDIRECT METHODS (Ref. 2)

The problem considered in Ref. 1 is investigated once more with reference to the class of slender, flat-top bodies of arbitrary longitudinal and tranversal contours. By means of the indirect methods of the calculus of variations, it is proved that the solutions obtained in Ref. 1 for the longitudinal contour are variational solutions.

5. INDIRECT METHODS (Ref. 5)

The problem of minimizing the drag of a slender, flat-top body of arbitrary longitudinal contour and semicircular transversal contour is considered under the assumption that the lift is given. The indirect methods of the calculus of variations are employed, and the necessary conditions to be satisfied by an optimum body are derived for arbitrary conditions imposed on the wetted area, the volume, the length, and the thickness. The particular cases treated are the following: (a) given lift, (b) given lift and thickness, (c) given lift and wetted area, (d) given lift and length, and (e) given lift and volume. For the first three cases, simple analytical solutions are found; for the last two, parametric solutions of a more complex nature are determined.

6. INDIRECT METHODS (Ref. 6)

The problem of maximizing the lift-to-drag ratio of a slender, flat-top body of arbitrary longitudinal contour and semicircular transversal contour is investigated under the assumption that the lift is unconstrained. Configurations are considered upon which two geometric constraints are imposed, that is, (a) given thickness and length, (b) given volume and length, and (c) given volume and thickness. For each case, the lift-to-drag ratio parameter, the thickness ratio parameter, and the optimum shape are presented as functions of a single similarity parameter involving the given quantities and the skin-friction coefficient.

7. SIMILARITY LAWS (Refs. 3 and 8)

In the previous investigations, the maximum lift-to-drag ratio was determined for certain particular combinations of constraints imposed on the lift, the pitching moment, the planform area, the frontal area, the wetted area, the volume, the length, and the thickness. Since the number of possible variational problems is practically without limit, economy of thought leads one to pose the following questions: (1) Is there any similarity law which permits one to determine the optimum longitudinal contour of a body of arbitrary transversal contour from the known optimum longitudinal contour of a reference body? and (2) Is there any similarity law which permits one to determine the optimum transversal contour of a body of arbitrary longitudinal contour from the known optimum transversal contour of a reference body? The answer to these questions can be found in Refs. 3 and 8 where two similarity laws are derived.

The SIMILARITY LAW FOR LONGITUDINAL CONTOURS permits one to determine the optimum longitudinal contour of a body of arbitrary transversal contour from the known optimum longitudinal contour of a reference body (a body of semi-circular cross section): the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants depending only on the prescribed transversal contour.

The SIMILARITY LAW FOR TRANSVERSAL CONTOURS permits one to determine the optimum transversal contour of a body of arbitrary longitudinal contour from the known optimum transversal contour of a reference body (a conical body): the aerodynamic and geometric quantities of the latter must be replaced by appropriate proportional quantities of the former, with the proportionality constants dependong only on the prescribed longitudinal contour.

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